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EXERCISES.

277

Calling Bernoulli's numbers B_1, B_2, . . . , prove the relation

$$B_1 - {}^{2r-1}C_2\,B_3 + {}^{2r-1}C_4\,B - \ldots + (-)^r . {}^{2r-1}C_{2r-4}\,B_{2r-3} = (-)^r . (2r+1)\,B_{2r-1},$$

where ${}^pC_q$ means the number of combinations of p things taken q at a time.

[*Frank Morley.*]

278

The sum of the cubes of the coefficients in the expansion of $(1 - x)^{2n}$ is $(-)^n\,(3n)!\,/\,(n!)^3$.

[*Frank Morley.*]

279

Let $f(X, Y) = 0$ be the equation of a curve in circular co-ordinates. Then it is seen at once that

$$(X - X_1)\frac{\partial f}{\partial X} = (Y - Y_1)\frac{\partial f}{\partial Y}$$

cuts f at the feet of the normals from (X_1, Y_1). Show that any asymptote of this curve cuts the asymptote of f at angles θ_r such that $\Sigma\,\cot\theta_r = 0$.

[*Frank Morley.*]

280

1. The centre locus

$$\begin{vmatrix} 0 & \alpha & \beta & c \\ \alpha & 0 & \gamma & b \\ \beta & \gamma & 0 & \alpha \\ n & m & l & 0 \end{vmatrix} = 0,$$

passes through the middle points of the sides of the fundamental triangle.

2. The fourth point of intersection of two such centre loci is the centre of the conic

$$(mn' - m'n)\,\beta\gamma + (nl' - n'l)\,\gamma\alpha + (lm' - l'm)\,\alpha\beta = 0.$$

[*R. H. Graves.*]

281

Find the fourth point of intersection of the nine-points circle, and the maximum ellipse inscribed in the fundamental triangle.

[*R. H. Graves.*]

282

The centre and three points do not determine a conic when two of the points are at the extremities of a diameter $(2k)$. [*R. H. Graves.*]

283

Let four circles, A, B, C, D, pass through a common point, and three straight lines through the same point; meet them again in the points A_1, B_1, C_1, D_1; A_2, B_2, C_2, D_2; A_3, B_3, C_3, D_3; then, distinguishing lines in opposite directions by opposite signs, will

$$\begin{vmatrix} A_1B_1 & A_1C_1 & A_1D_1 \\ A_2B_2 & A_2C_2 & A_2D_2 \\ A_3B_3 & A_3C_3 & A_3D_3 \end{vmatrix} = 0.$$

YALE PRIZE PROBLEMS.

FRESHMAN.

284

If the angular elevation of the summits of two spires (which appear in a straight line) is α, and the angular depressions of their reflections in a lake, h feet below the point of observation, are β and γ, then the horizontal distance between the spires is

$$2h \cos^2\alpha \sin(\beta - \gamma) \operatorname{cosec}(\beta - \alpha) \operatorname{cosec}(\gamma - \alpha).$$

285

Let a, b, c be the sides of a plane triangle, and α, β, γ the lines bisecting the angles and terminating in the opposite sides, then will

$$\frac{\alpha\beta\gamma}{abc} = \frac{4(a+b+c)}{(a+b)(b+c)(c+a)} \times \text{area of triangle.}$$

286

The squares of the sides of a plane triangle added to the squares of the radii of the four circles respectively touching these sides is equal to four times the square of the diameter of the circumscribing circle.

287

Factor the expression,

$$abc(x^3 + y^3 + z^3) + (a^2b + b^2c + c^2a)(x^2y + y^2z + z^2x)$$
$$+ (a^2c + c^2b + b^2a)(x^2z + z^2y + y^2x) + (a^2 + b^3 + c^3)xyz + 3abcxyz.$$

288

If $\varkappa$, λ, μ, ν be any four angles, and if

$$\varkappa + \lambda + \mu + \nu = 2\sigma,$$

prove

$$\begin{aligned}&\sin\varkappa \sin\lambda \sin\mu \sin\nu + \cos\varkappa \cos\lambda \cos\mu \cos\nu \\ &\quad = \sin(\sigma - \varkappa)\sin(\sigma - \lambda)\sin(\sigma - \mu)\sin(\sigma - \nu) \\ &\quad + \cos(\sigma - \varkappa)\cos(\sigma - \lambda)\cos(\sigma - \mu)\cos(\sigma - \nu).\end{aligned}$$

289

The points P, Q, R, S, T, U are the vertices of a regular hexagon inscribed in a circle with centre O. Join PR cutting OQ in A; AS, cutting OR in B; BT, cutting OS in C, etc. Prove that OA, OB, OC, OD, etc., are proportional to the reciprocals of the natural numbers 1, 2, 3, 4, etc.

290

Let A, B, C be any three points in space; find the locus of a point whose distances to these three points are in the ratio of three given lines.

SOPHOMORE.

291

A regular tetrahedron and a regular octahedron are inscribed in the same sphere; compare the radii of the spheres which can be inscribed in the two solids.

292

Two ports are in the same parallel of latitude, their common latitude being l and their difference of longitude 2λ; find the saving of distance in sailing from one to the other on a great circle, instead of sailing due east or west.

293

If a triangle be cut out of paper and doubled over so that the crease passes through the centre of the circumscribing circle and the angle A, the area of the triangular double-part will be

$$\tfrac{1}{2}b^2 \sin^2 C \cos C \operatorname{cosec}(2C - B) \sec(C - B);$$

the angle C being greater than B.

294

Find the equation on the developed surface of a cone of the section line made by the intersection of a plane with the cone.

295

Find the axes of the elliptic section which the plane passing through the centre of a given oblate spheroid, and through two points whose geodetic latitudes and longitudes on the surface of the spheroid are given, makes with the spheroid.

296

(1) A complete plane quadrangle $ABCD$ has six sides AB, AC, . . . which intersect an arbitrary line s in six points P_{AB}, P_{AC}, ... Show (*a*) that a quadrangle may be constructed which shall in this way correspond to *any* five collinear points to which such a notation has been at random but permanently affixed, and (*b*) that all such quadrangles lead to the same sixth point on the line.

(2) A complete quadrilateral $k_1k_2k_3k_4$ has six vertices k_1k_2, k_1k_3, . . . and three diagonals $d_{12\cdot34} = (k_1k_2)\,(k_3k_4)$, ... A transversal s cuts the sides in the points K_1, K_2, K_3, K_4, and the diagonals in the, say, *diagonal* points $D_{12\cdot34}$, $D_{13\cdot42}$, $D_{14\cdot23}$. Show (by a double application of (1, *b*) in each case) that these seven collinear points *with this notation* are *determined*

(*c*) by the four points K and any one of the diagonal points D,

(*d*) by any three of the four points K and any two of the diagonal points D;

i. e. (*ca*) a quadrilateral may be constructed which shall in the way defined correspond to *any* five collinear points to which the notation (*c*) has been at random (but, during the consideration of case (*c*), permanently) affixed, and (*cb*) all such quadrilaterals lead to the same two remaining points; and similarly for case (*d*).

JUNIOR.

297

Six points A, B, C, L, M, N are taken on a sphere, and the great circles AL, AM, AN, BL, BM, BN, CL, CM, CN are drawn. Prove that if AL, BM, CN meet in a point, and AM, BN, CL meet in a point, then AM, BL, CM will also meet in a point.

298

R being the radius of the circle circumscribed about a triangle, r the radius of the circle inscribed in it, and s the half sum of the sides of the triangle, the radii of the escribed circles are the roots of the equation

$$(x^2 + s^2)\,(x - r) = 4Rx^2.$$

299

Given in a plane two arbitrary points P, Q, and on any circle two arbitrary points A, B; let the four lines PA, PB, QA, QB be drawn, cutting the circle in four additional points P_a, P_b, Q_a, Q_b. Prove that the join-lines P_aQ_b, P_bQ_a intersect at some point on the line joining PQ.

300

Given two triangles, each reciprocally polar to the other with respect to a conic. Show that the three lines joining corresponding vertices are concurrent

SENIOR.

301

A solid is in equilibrium under the influence of four forces, of which the directions and magnitudes are given. Find the ratios of the six perpendicular distances between the lines of force.

302

Power is transmitted from one pulley to another at the same level by an endless cable, which is considerably longer than is necessary to encircle the pullies, and is to be regarded as perfectly flexible. Discuss the two loops of the cable when the system is in a state of steady motion, and show how the rate of transmission of power may be determined from the form of the loops and the velocity of the motion.

303

Uniform spheres are piled as compactly as possible. Describe the solid figures obtained by passing tangent planes through every point of contact. Give the ratio of the volume of a pile containing an infinite number of spheres to the aggregate volume of the spheres.

304

Discuss the equilibrium of an open conical cup floating in water.